

## CLAIMS

- 1 1. A method for trading in a financial derivative of an  
2 underlying asset, comprising:  
3 determining a trend predictive of a future value of  
4 the asset and a predicted variance of the future value;  
5 responsive to the trend and the variance,  
6 calculating a density function indicative of a  
7 probability distribution of the value at a first time in  
8 the future;  
9 based on the density function at the first time,  
10 recalculating the density function to find the  
11 probability distribution of the value at a second time,  
12 subsequent to the first time; and  
13 making a trading decision with regard to the  
14 derivative of the asset based on the density function.
- 1 2. A method according to claim 1, wherein recalculating  
2 the density function comprises finding a change in the  
3 density function due to a stochastic process governing  
4 the value of the asset.
- 1 3. A method according to claim 2 wherein finding the  
2 change in the density function comprises integrating a  
3 random variable representative of the stochastic process  
4 over the density function at the first time.
- 1 4. A method according to claim 3, wherein the random  
2 variable has a plurality of discrete values with a normal  
3 probability distribution.
- 1 5. A method according to claim 4, wherein the random  
2 variable comprises a superposition of delta functions.
- 1 6. A method according to claim 3, wherein integrating  
2 the density function comprises computing an integrated

3 value of the function at each of a plurality of grid  
4 points in a coordinate space, wherein the value of the  
5 asset is represented by a coordinate in the space.

1 7. A method according to claim 6, wherein computing the  
2 integrated value comprises, for each of the plurality of  
3 grid points:

4 finding one or more corresponding points in the  
5 coordinate space at the first time, such that in a time  
6 step from the first time to the second time, the random  
7 variable makes a transition from the one or more  
8 corresponding points to the grid point at the second  
9 time; and

10 determining the function at the grid point at the  
11 second time by summing over the density function at the  
12 corresponding points.

1 8. A method according to claim 6, wherein recalculating  
2 the density function comprises interpolating the density  
3 function intermediate the grid points so as generate a  
4 smooth function over a selected range of calculation.

1 9. A method according to claim 8, wherein computing the  
2 integrated value comprises computing the value and a  
3 first derivative of the density function at the second  
4 time, and wherein interpolating the density function  
5 comprises fitting polynomial functions between the grid  
6 points so as to match the value and the first derivative  
7 of the density function computed at each of the grid  
8 points.

1 10. A method according to claim 9, and comprising  
2 recalculating the density function at a third time,  
3 subsequent to the second time, using the polynomial

4 functions fitted between the grid points at the second  
5 time.

1 11. A method according to claim 1, wherein recalculating  
2 the density function comprises iteratively recalculating  
3 the density function at each of a plurality of times from  
4 the first time up to a final time.

1 12. A method according to claim 1, wherein determining  
2 the trend comprises finding a multivariate trend with  
3 respective to a plurality of related variables that  
4 include the value of the asset.

1 13. A method according to claim 12, wherein determining  
2 the variance comprises finding a covariance matrix with  
3 respect to the plurality of related variables.

1 14. A method according to claim 12, wherein the density  
2 function comprises a multivariate function, based on at  
3 least some of the plurality of related variables, in a  
4 multidimensional coordinate space.

1 15. A method according to claim 1, wherein making the  
2 trading decision comprises computing an expected value of  
3 the asset based on the density function.

1 16. A method according to claim 1, wherein making the  
2 trading decision comprises computing an expected yield of  
3 the financial derivative based on the density function.

1 17. A method according to claim 1, wherein making the  
2 trading decision comprises deciding whether to carry out  
3 a transaction in the financial derivative at a given  
4 transaction price.

1 18. A method according to claim 1, wherein the financial  
2 derivative comprises an option exercisable at any of a

3 plurality of points in time, and wherein making the  
4 trading decision comprises determining at which of the  
5 points to exercise the option.

1 19. A method according to claim 18, wherein determining  
2 at which of the points to exercise the option comprises  
3 calculating a strategy function, and deciding whether to  
4 exercise the option by comparing a current value of the  
5 asset to a value of the strategy function at one or more  
6 of the points in time.

1 20. A method according to claim 1, wherein the  
2 derivative comprises a path-dependent option, and wherein  
3 recalculating the density function comprises computing a  
4 path-dependent density function.

1 21. A method according to claim 20, wherein computing  
2 the path-dependent density function comprises finding a  
3 cumulative density function indicative of a  
4 path-dependent probability distribution of a value of the  
5 option.

1 22. Apparatus for trading in a derivative of an  
2 underlying asset, comprising a decision processor, which  
3 is adapted, responsive to a trend predictive of a future  
4 value of the asset and to a predicted variance of the  
5 future value, to calculate a density function indicative  
6 of a probability distribution of the value at a first  
7 time in the future and, based on the density function at  
8 the first time, to recalculate the density function to  
9 find the probability distribution of the value at a  
10 second time, subsequent to the first time, and to provide  
11 an output for use in making a trading decision with  
12 regard to the derivative of the asset based on the  
13 density function.

1 23. Apparatus according to claim 22, wherein the  
2 processor is adapted to find a change in the density  
3 function due to a stochastic process governing the value  
4 of the asset.

1 24. Apparatus according to claim 23, wherein the  
2 processor is adapted to find the change in the density  
3 function by integrating a random variable representative  
4 of the stochastic process over the density function at  
5 the first time.

1 25. Apparatus according to claim 24, wherein the random  
2 variable has a plurality of discrete values with a normal  
3 probability distribution.

1 26. Apparatus according to claim 24, wherein the  
2 processor is adapted to compute an integrated value of  
3 the function at each of a plurality of grid points in a  
4 coordinate space, wherein the value of the asset is  
5 represented by a coordinate in the space.

1 27. Apparatus according to claim 26, wherein the  
2 processor is adapted to interpolate the density function  
3 intermediate the grid points so as generate a smooth  
4 function over a selected range of calculation.

1 28. Apparatus according to claim 27, wherein the  
2 processor is adapted to interpolate the density function  
3 by fitting polynomial functions between the grid points  
4 so as to match a value and a first derivative of the  
5 density function computed by the processor at each of the  
6 grid points.

1 29. Apparatus according to claim 22, wherein the  
2 processor is adapted to iteratively recalculate the

3 density function at each of a plurality of times from the  
4 first time up to a final time.

1 30. Apparatus according to claim 22, wherein the trend  
2 comprises finding a multivariate trend with respect to  
3 a plurality of related variables that include the value  
4 of the asset.

1 31. Apparatus according to claim 22, wherein the  
2 processor is adapted to compute an expected value of the  
3 asset based on the density function.

1 32. Apparatus according to claim 22, wherein the  
2 processor is adapted to compute an expected yield of the  
3 financial derivative based on the density function.

1 33. Apparatus according to claim 22, wherein the  
2 financial derivative comprises an option exercisable at  
3 any of a plurality of points in time, and wherein the  
4 processor is adapted to compute a strategy function for  
5 use in determining at which of the points to exercise the  
6 option.

1 34. Apparatus according to claim 1, wherein the  
2 derivative comprises a path-dependent option, and wherein  
3 the processor is adapted compute a path-dependent density  
4 function.

1 35. A computer software product for use in trading in a  
2 derivative of an underlying asset, the product comprising  
3 a computer-readable medium in which program instructions  
4 are stored, which instructions, when read by a computer,  
5 cause the computer, responsive to a trend predictive of a  
6 future value of the asset and to a predicted variance of  
7 the future value, to calculate a density function  
8 indicative of a probability distribution of the value at

9 a first time in the future and, based on the density  
10 function at the first time, to recalculate the density  
11 function to find the probability distribution of the  
12 value at a second time, subsequent to the first time, and  
13 to provide an output for use in making a trading decision  
14 with regard to the derivative of the asset based on the  
15 density function.

1 36. A product according to claim 35, wherein the  
2 instructions cause the computer to recalculate the  
3 density function by finding a change in the density  
4 function due to a stochastic process governing the value  
5 of the asset.

1 37. A product according to claim 36, wherein the  
2 instructions cause the computer to find the change in the  
3 density function by integrating a random variable  
4 representative of the stochastic process over the density  
5 function at the first time.

1 38. A product according to claim 37, wherein the random  
2 variable has a plurality of discrete values with a normal  
3 probability distribution.

1 39. A product according to claim 37, wherein the  
2 instructions cause the computer to compute an integrated  
3 value of the function at each of a plurality of grid  
4 points in a coordinate space, wherein the value of the  
5 asset is represented by a coordinate in the space.

1 40. A product according to claim 39, wherein the  
2 instructions further cause the computer to interpolate  
3 the density function intermediate the grid points so as  
4 generate a smooth function over a selected range of  
5 calculation.

1 41. A product according to claim 40, wherein the  
2 instructions cause the computer to determine the value  
3 and a first derivative of the density function at the  
4 second time, and to interpolate the density function by  
5 fitting polynomial functions between the grid points so  
6 as to match the value and the first derivative of the  
7 density function computed at each of the grid points.

1 42. A product according to claim 35, wherein the  
2 instructions cause the computer to iteratively  
3 recalculating the density function at each of a plurality  
4 of times from the first time up to a final time.

1 43. A product according to claim 35, wherein the trend  
2 comprises a multivariate trend with respect to a  
3 plurality of related variables that include the value of  
4 the asset.

1 44. A product according to claim 43, wherein the density  
2 function comprises a multivariate function, based on at  
3 least some of the plurality of related variables, in a  
4 multidimensional coordinate space.

1 45. A product according to claim 35, wherein the  
2 instructions cause the computer to determine an expected  
3 value of the asset based on the density function.

1 46. A product according to claim 35, wherein the  
2 instructions cause the computer to determine an expected  
3 yield of the financial derivative based on the density  
4 function.

1 47. A product according to claim 35, wherein the  
2 financial derivative comprises an option exercisable at  
3 any of a plurality of points in time, and wherein the  
4 instructions cause the computer to calculate a strategy



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5 function for determining at which of the points to  
6 exercise the option.

1 48. A product according to claim 35, wherein the  
2 derivative comprises a path-dependent option, and wherein  
3 the instructions cause the computer to determine a  
4 path-dependent density function.